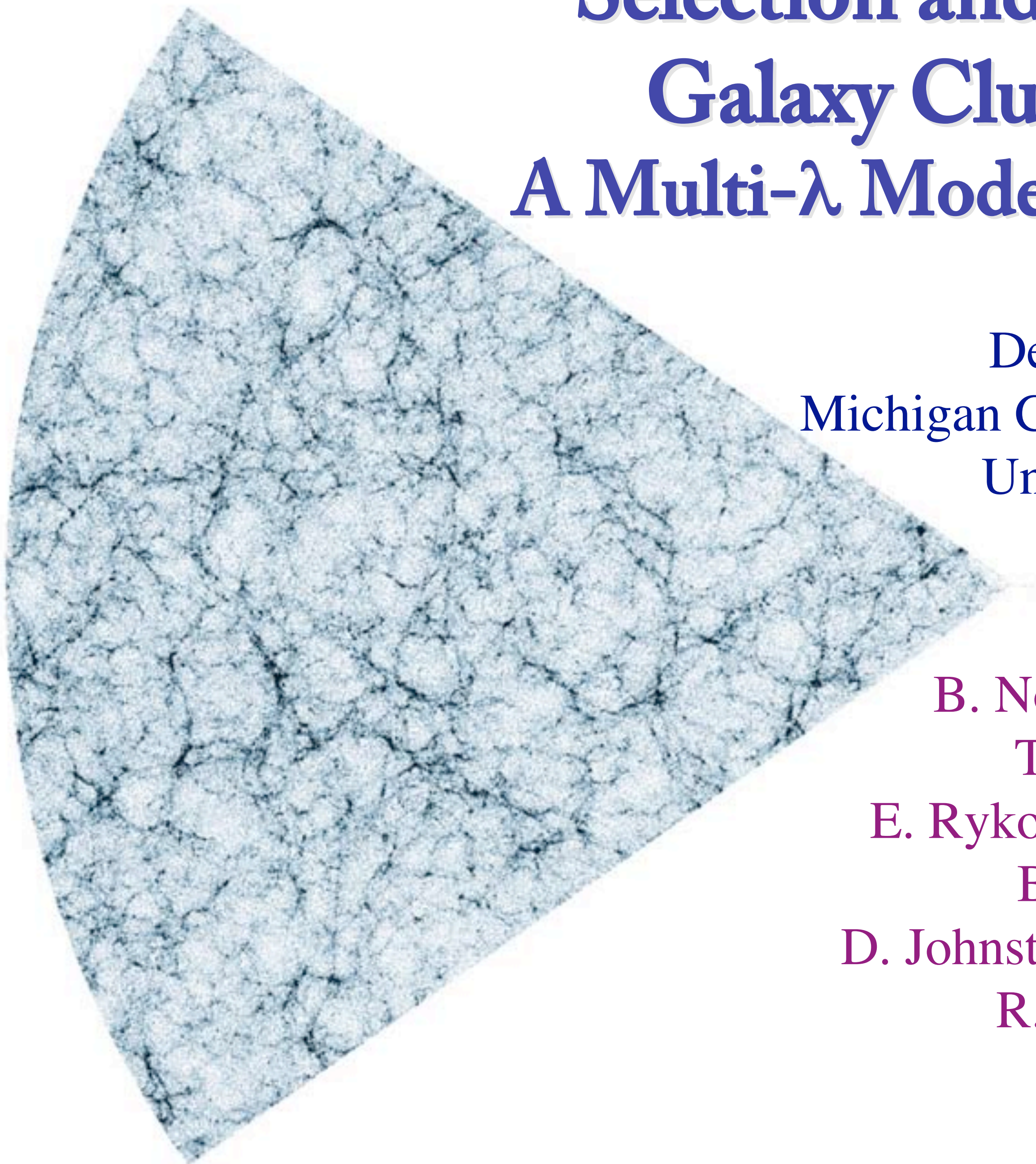
Selection and Covariance in Galaxy Cluster Surveys: A Multi-λ Model for Local Counts



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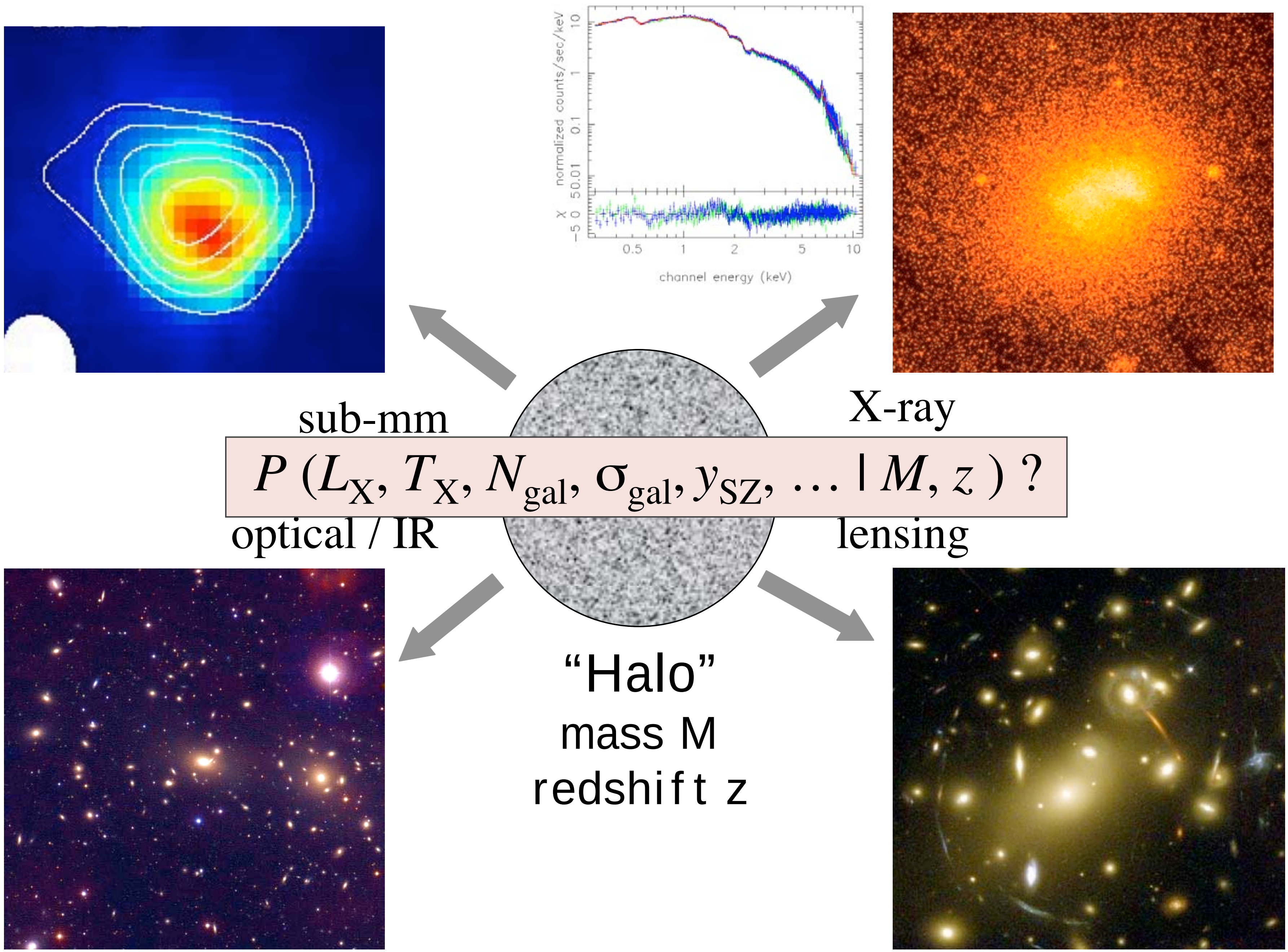
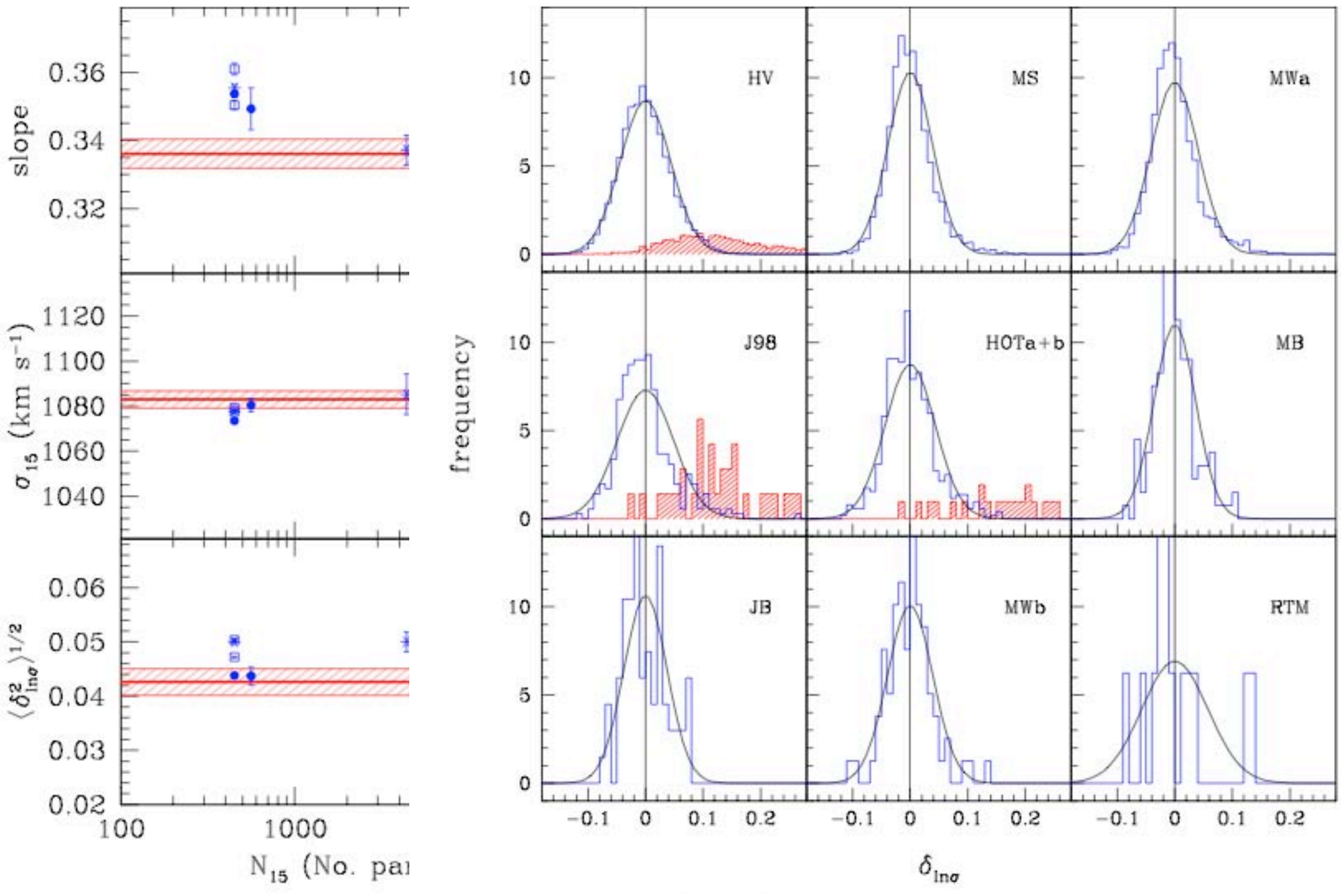

normalized counts/sec/keV
channel energy (keV)
sub-mm
X-ray
$P\,(L_X, T_X, N_{gal}, \sigma_{gal}, y_{SZ}, \ldots \mid M, z\,)$?
optical / IR
lensing
"Halo"
mass M
redshift z

Precise determination of DM virial scaling

Evrard et al 2008

virial relation in dark matter from 18 large N-body sims, 6 codes



A power-law + scatter model for multiple observables

- For $i^{\rm th}$ proxy, mean behavior of $s_i = \ln(S_i)$ is linear in $\ln M$ w/ slope m_i. For N such signals

$$\bar{\mathbf{s}}(\mu, z) = \mathbf{m}(z)\mu + \mathbf{b}(z)$$

$$\mu = \ln M$$

- assume a log-normal joint likelihood about the mean

$$p(\mathbf{s} \mid \mu, z) = \frac{1}{(2\pi)^{N/2} |\Psi|^{1/2}} \exp[-\tfrac{1}{2}(\mathbf{s} - \bar{\mathbf{s}})' \Psi^{-1} (\mathbf{s} - \bar{\mathbf{s}})]$$

where Ψ is the ***covariance in signals*** at fixed mass and epoch

$$\Psi_{ij} = \left\langle \left(s_i - \bar{s}_i(\mu, z)\right)\left(s_j - \bar{s}_j(\mu, z)\right) \right\rangle$$

Local model for multi-observable counts (the ***s**-function*)

- locally power-law mass function $dp = n(\mu)\, dV$

$$n(\mu) = A\exp(-\alpha\mu)$$

- convolve with log-normal likelihood for **s** to find the

joint property space density

$$n(\mathbf{s}) = \frac{A\Sigma}{(2\pi)^{(N-1)/2}|\Psi|^{1/2}} \exp\left[-\frac{1}{2}\left(\mathbf{s}'\Psi^{-1}\mathbf{s} - \frac{\overline{\mu}^2(\mathbf{s})}{\Sigma^2}\right)\right]$$

where Σ^2 is the mass variance, and $\underline{\mu}$ is the log-mean mass

$$\Sigma^2 = \left(\mathbf{m}'\Psi^{-1}\mathbf{m}\right)^{-1}$$

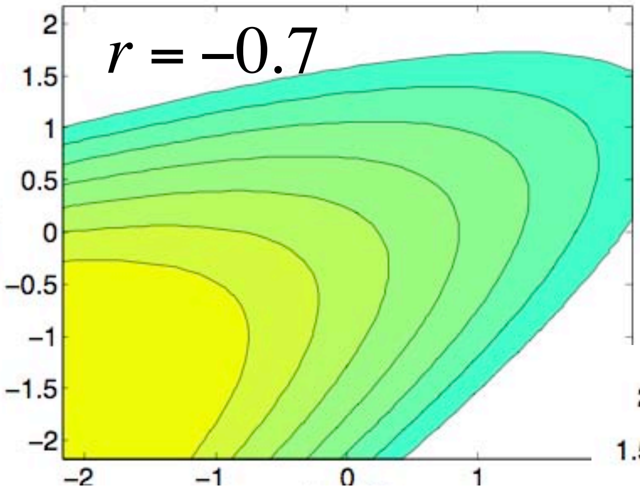
$$\overline{\mu}(\mathbf{s}) = \frac{\mathbf{m}'\Psi^{-1}\mathbf{s}}{\mathbf{m}'\Psi^{-1}\mathbf{m}} - \alpha\Sigma^2$$

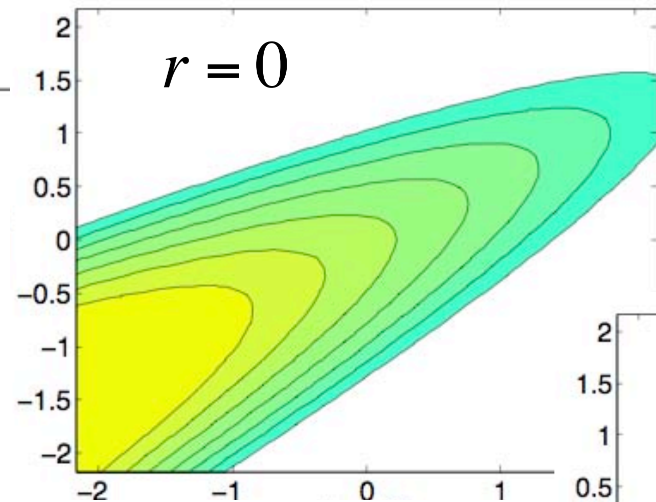
Note: **b**=0 or **s** = **s**–**b**(z) above.

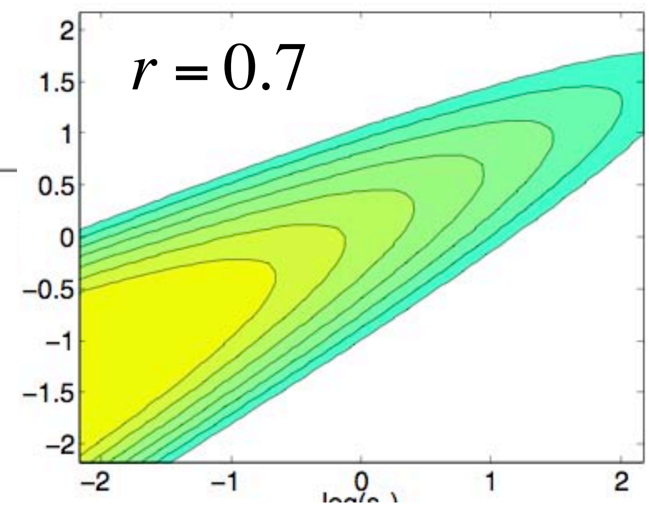
Explicit 2D number counts: contours of log(n($\mathbf{s}$))

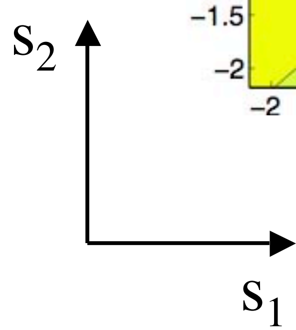
$$\alpha = 3,\ m = [1.6\ 1.0],\ \sigma = [0.5\ 0.5]$$

$$r = \left\langle \frac{\delta s_1}{\sigma_1} \frac{\delta s_2}{\sigma_2} \right\rangle$$









Mass selection properties

- Bayes' theorem => Gaussian expectation for mass selection

$$p(\mu \mid \mathbf{s}) = \frac{1}{\sqrt{2\pi}\Sigma} \exp\left[-\frac{\left(\mu - \bar{\mu}(\mathbf{s})\right)^2}{2\Sigma^2} \right]$$

with *biased mean*

$$\bar{\mu}(\mathbf{s}) = \bar{\mu}_0(\mathbf{s}) - \alpha\Sigma^2$$

$$\bar{\mu}_0(\mathbf{s}) = \Sigma^2 \left(\mathbf{m}' \Psi^{-1} \mathbf{s} \right)$$

selection bias from asymmetric scatter off a steep MF

inverse of input log-mean relation

Good news : bias in mass scales as the variance

Bad news : high-end mass function is steep, $\alpha \sim 3$

Explicit 2D example

$$\Psi = \begin{bmatrix} \sigma_1^2 & r\sigma_1\sigma_2 \\ r\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

correlation coefficient

$r \in (-1,1)$

- define equivalent scatter in mass $\sigma_{\mu i} = \sigma_i / m_i$

- the log-mass variance is a harmonic mixture

$$\Sigma^{-2} = \frac{\sigma_{\mu 1}^{-2} + \sigma_{\mu 2}^{-2} - 2r\sigma_{\mu 1}^{-1}\sigma_{\mu 2}^{-1}}{1 - r^2}$$

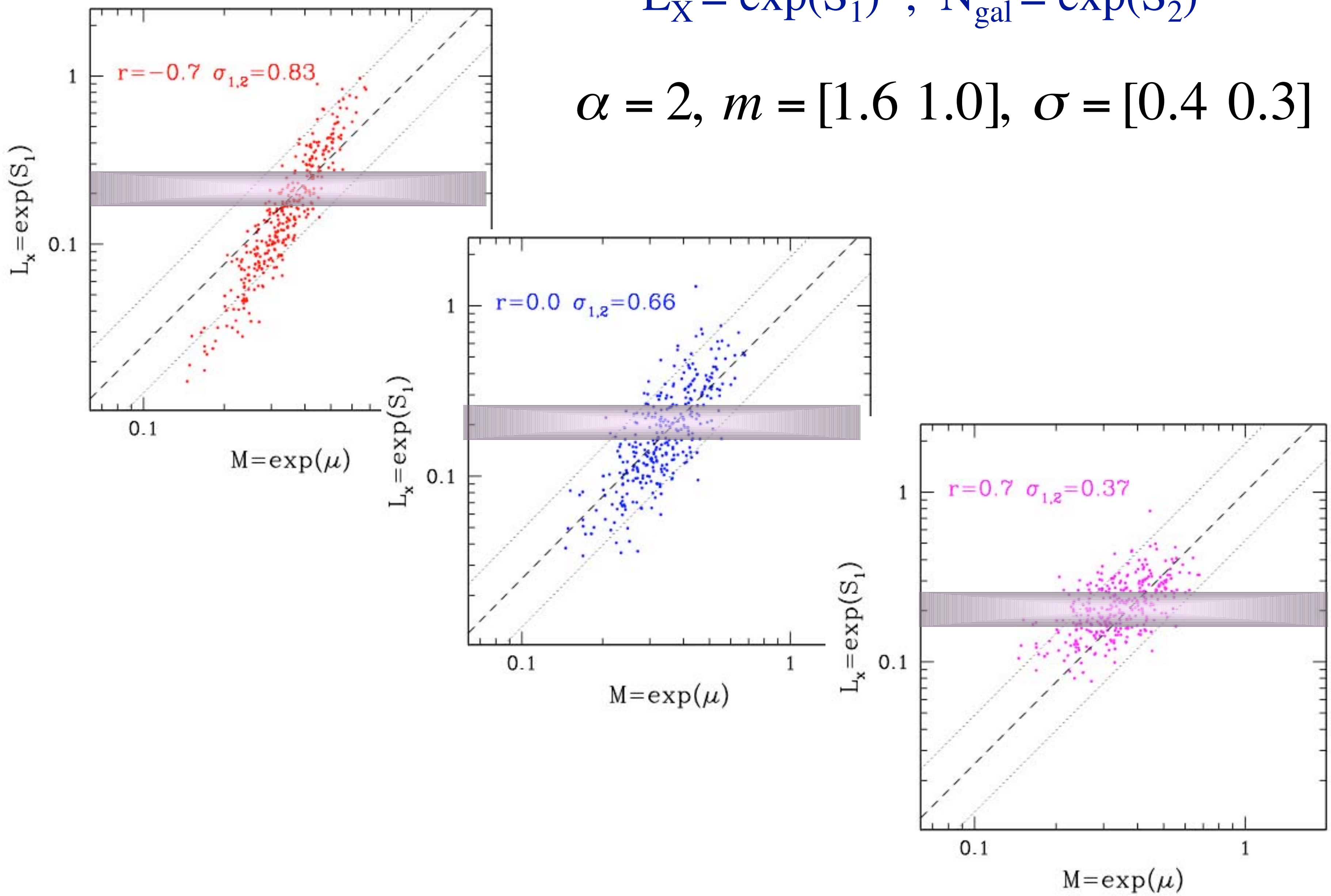
To minimize scatter and bias in mass, we want:

- small intrinsic scatter
- steep mass relation ($m_i > 1$)
- anti-correlated signals (if $m_i > 0$)

S$_1$–M pairs for a fixed S$_2$ bin

$L_X = \exp(S_1)$; $N_{gal} = \exp(S_2)$

$$\alpha = 2,\ m = [1.6\ 1.0],\ \sigma = [0.4\ \ 0.3]$$

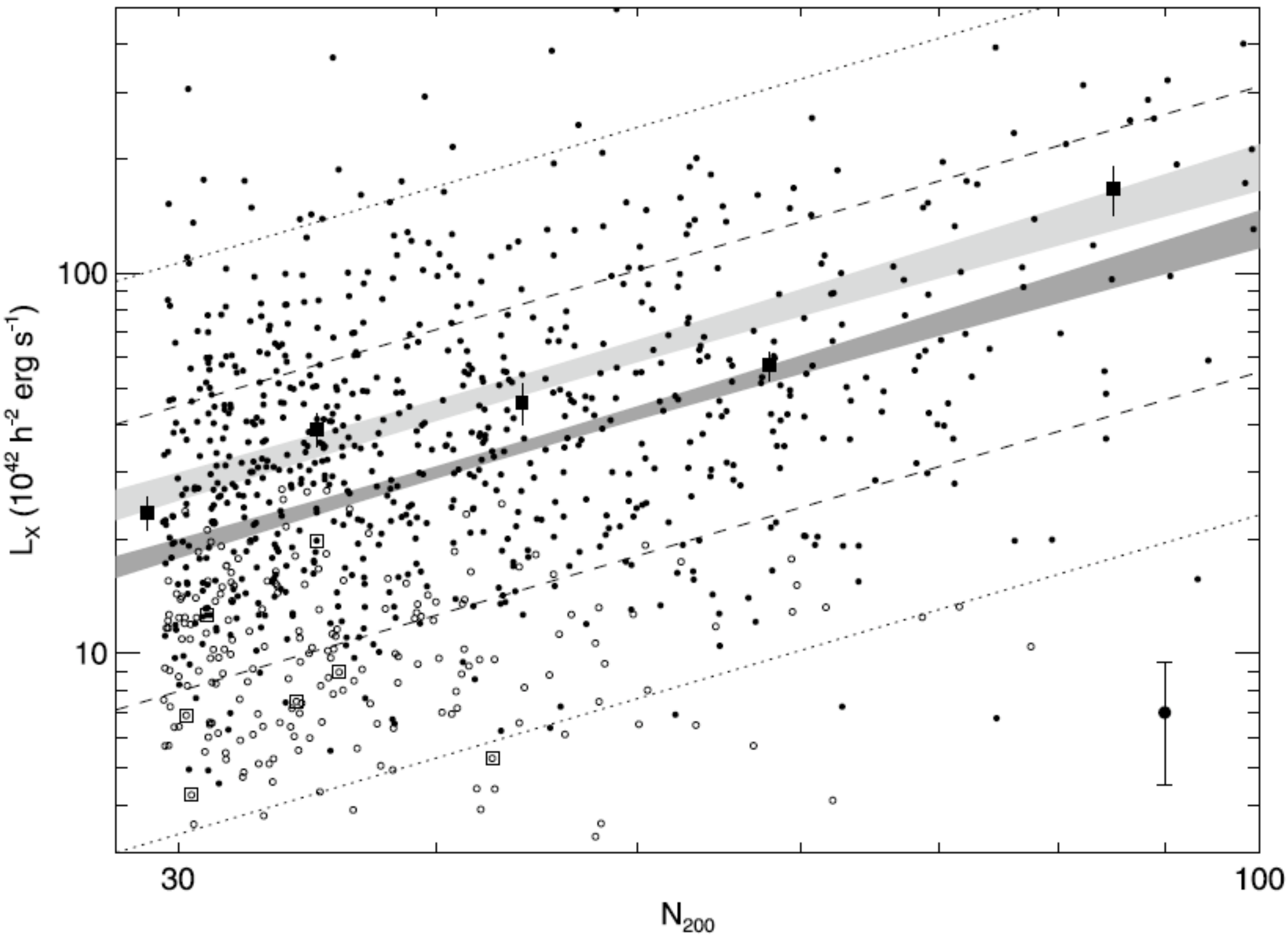


L_X–N_{gal} scatter measurement

17,000 SDSS maxBCG clusters with RASS detections/upper limits

$$\sigma_{\ln L_X, N_{gal}} = 0.83 \pm 0.03$$

Rykoff et al 2008a



L_X–M expectations for N_{gal} binned data

Rykoff et al 2008b

$$\ell = \ln L_X \;;\; \sigma_{\mu,\ell} = \sigma_\ell / m_\ell \qquad \nu = \ln N_{gal} \;;\; \sigma_{\mu,\nu} = \sigma_\nu / m_\nu$$

- log-mean behavior of binned data with mass

$$\bar{\ell}(\nu) = m_\ell \left(\bar{\mu}(\nu) + \alpha(\mu) r \sigma_{\mu,\ell} \sigma_{\mu,\nu} \right)$$

- implied slope of scaling with mean mass may be biased

$$d\bar{\ell} / d\bar{\mu} = m_\ell + (r \sigma_{\mu,\ell} \sigma_{\mu,\nu}) d\alpha / d\mu$$

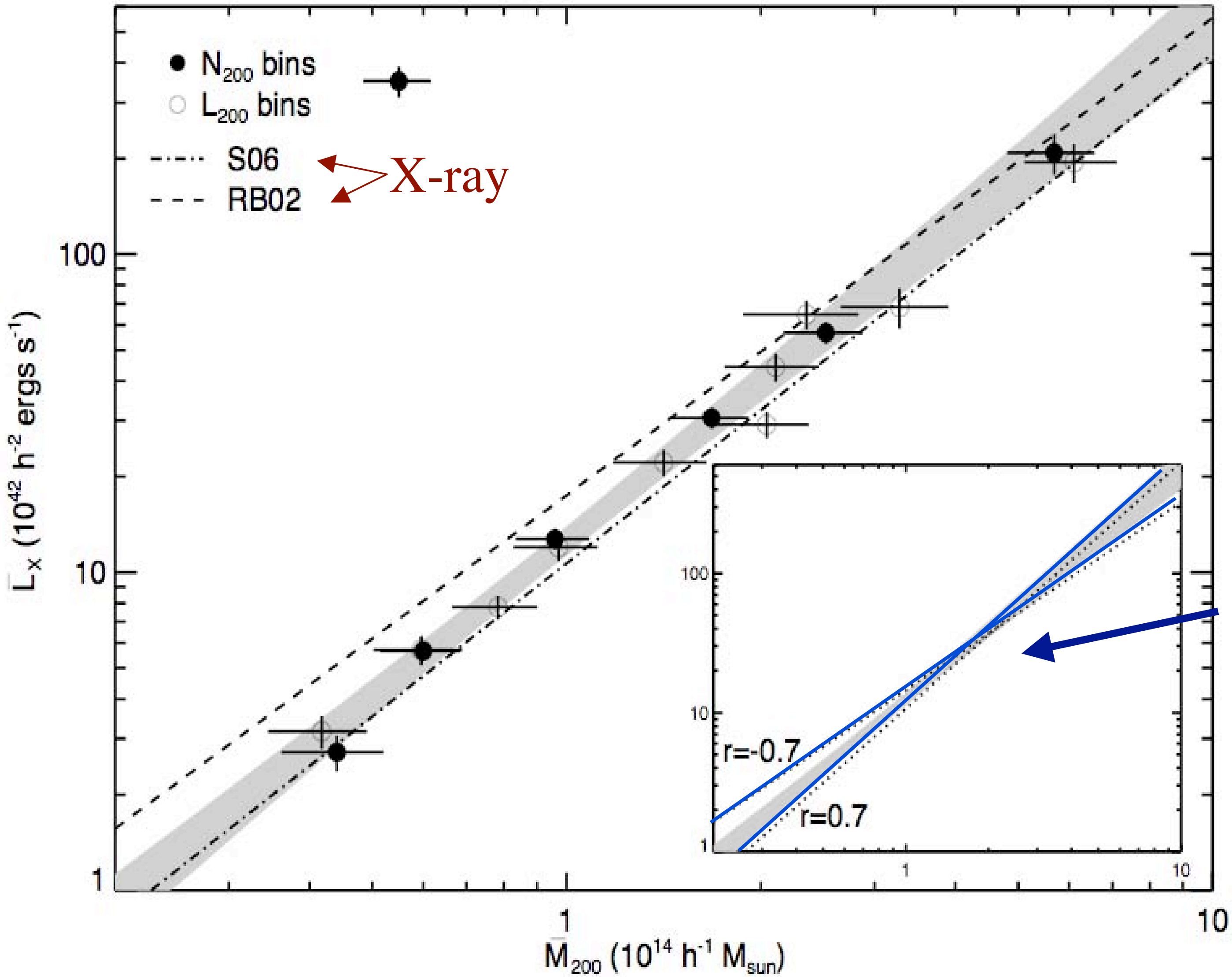
- variance is sensitive only to signal correlation

$$\sigma^2_{\ell,\nu} = m_\ell^2 \left(\sigma^2_{\mu,\ell} + \sigma^2_{\mu,\nu} - 2r \sigma_{\mu,\ell} \sigma_{\mu,\nu} \right)$$

L_X–M from maxBCG sample

Johston et al 2007
Rykoff et al 2008b

M_{200} from weak lensing, L_X from RASS, in fixed N_{gal} bins



Good agreement between X-ray and optically selected samples

Non-zero optical-Xray correlation can ***tilt*** N_{gal}-binned relation due to running of MF slope $\alpha(M)$.

magnitude scales with L_X-N_{gal} covariance

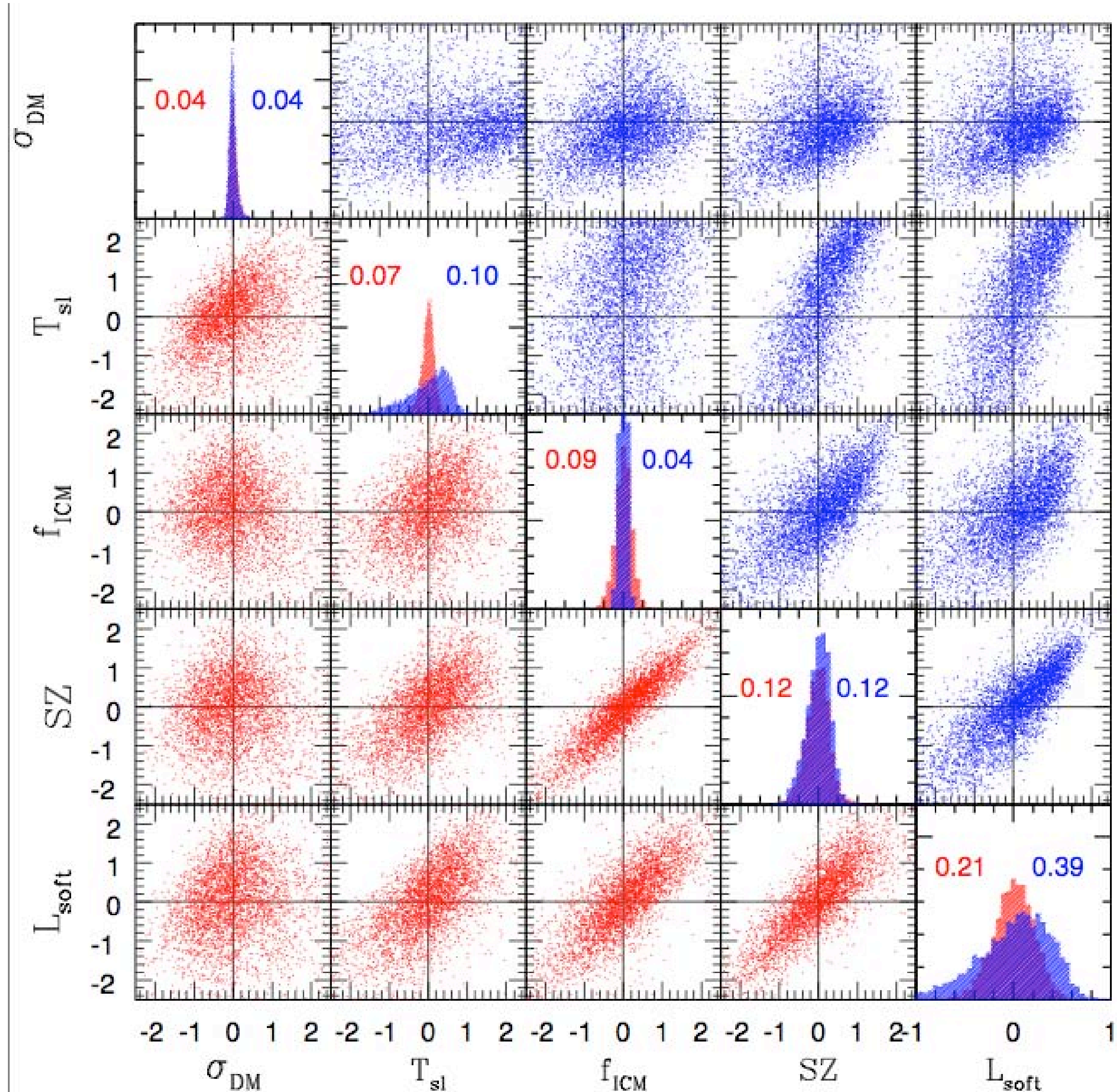
Is a power-law + multivariate Gaussian generic?

with Lorena Gazzola,
F. Pearce (Nottingham)

Millennium Simulation: Gadget2 with gas under two physical treatments:
- – preheating
- – gravity only

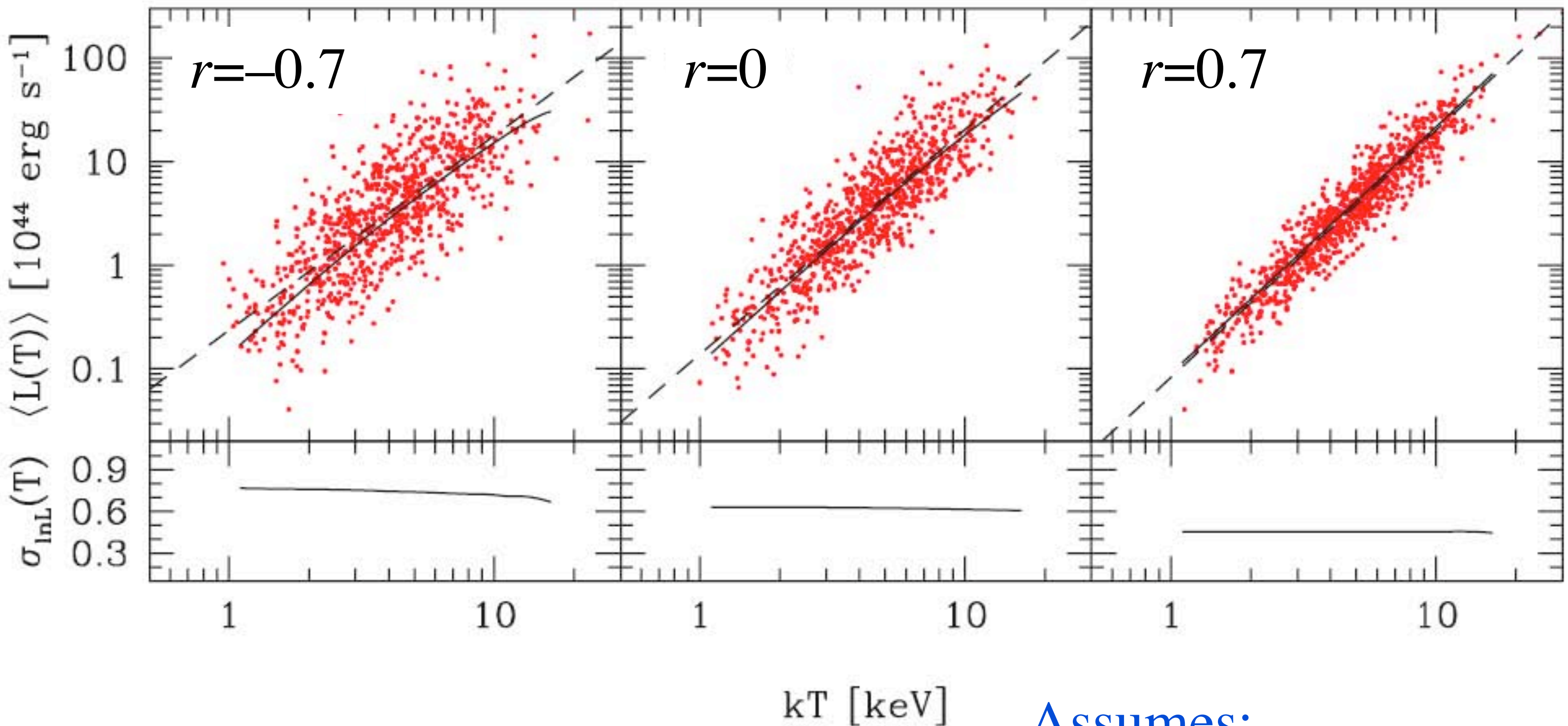
Covariance in ~3000 halos at z=0 with $M_{200} > 3x10^{13}$ Msun/h

Stanek et al, in prep



Local L-T relation: low-hanging covariant fruit?

Nord et al 2008



Predicted relations for a local X-ray flux-limited sample:

$f_{0.5\text{-}2\ \mathrm{keV}} > 3\mathrm{x}10^{-12}$ erg/s/cm^2

Assumes:

$\sigma_{\ln L,M} = 0.6$

$\sigma_{\ln T,M} = 0.1$

Need better empirical constraints on scatters & slopes.

Selection & Characterization: How to combine approaches?

Method	slope / scatter	mass scatter	blended fraction	comment
Optical	1.0 ± 0.2 / 0.1–0.5 ?	0.1–0.5 ?	5–20%	f_{blend} is likely to be z-dependent
SZ	1.6 ± 0.2 / 0.1–0.2 ?	0.06–0.12 ?	5–20% ?	ditto above, + no published detections
X-ray	1.6 ± 0.1 / 0.6 ± 0.1	0.37 ± 0.05	< 5%	Stanek etal scatter may be high

- SZ + optical will be done jointly (SPT + DES)

Use X-rays to characterize these detections?

+ mostly source photons

+ more clusters with well-measured T_X (compared to blind)

– timing: SZ source lists not yet available

Summary

- cluster survey analysis requires understanding of mass proxies

  basic halo model: power-law mean + log-normal covariance $p(\mathbf{s}|\mu)$

  fixed **s** selects log-normal *M* dist'n with mean biased by $\alpha\Sigma^2$

  (co-)variance needs to be understood

** Apparent variation in the mass scale will bias best-fit cosmology.*

** Variance in the proxy-mass relation will bias mass selection.*

- value of multiple cluster measures

  improved mass selection, understand covariance (physics)

- role of simulations

  test robustness of PL+log-normal covariance model

  selection function from mock survey skies (line-of-sight blending)

- role of XMM?

  discovery or characterization? mix of both!